Geometry in Patch Based Non-Local Denoising Algorithms

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I. Motivation

Techniques for denoising natural images have seen incredible advances over the past several decades, with the current state of the art using non-local patch based algorithms. Recent studies of denoising techniques have shown that we are not likely to do much better than the current state of the art when using image data alone.

Even the most state of the art algorithm, BM3D [3], has its limitations. For example, the denoised result in Fig. 1 shows smoothing effects in the flatter regions of the image that cause a loss of detail in the tiles, the clouds, and the edges of the roof.

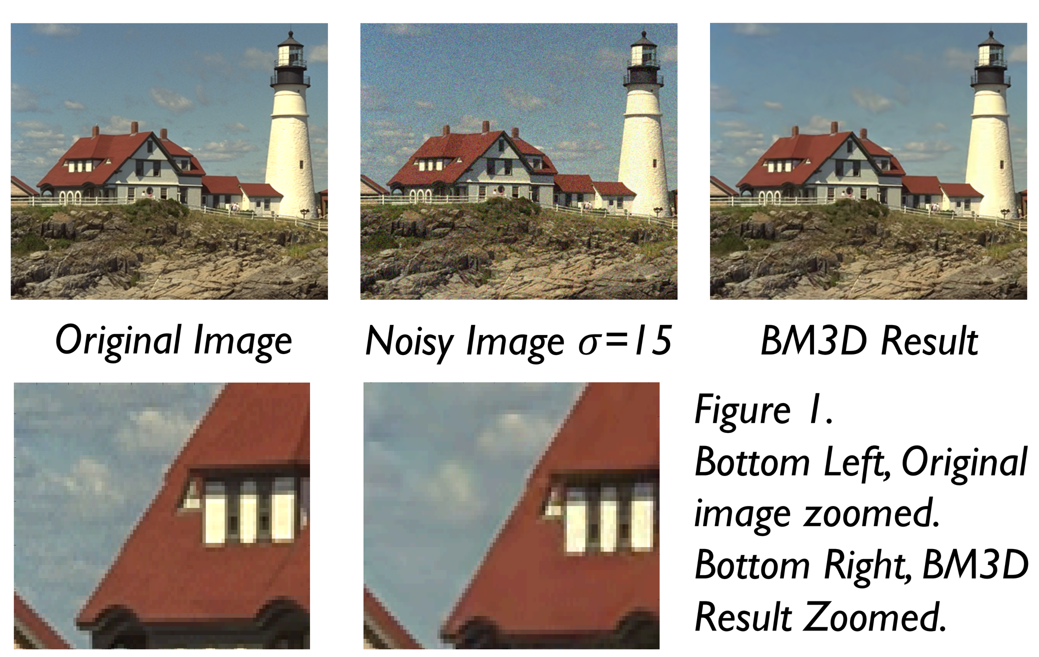


Figure 1. Bottom Left, Original image zoomed. Bottom Right, BM3D Result Zoomed.

II. Background

A noisy image y can be modeled by considering

$$y = x + n,$$

where x is the clean image and n is a random noise vector with entries that are independent and identically distributed according to a Gaussian with zero mean and variance σ^2. The goal of image denoising is to estimate a clean image $\hat{y}$ from y.

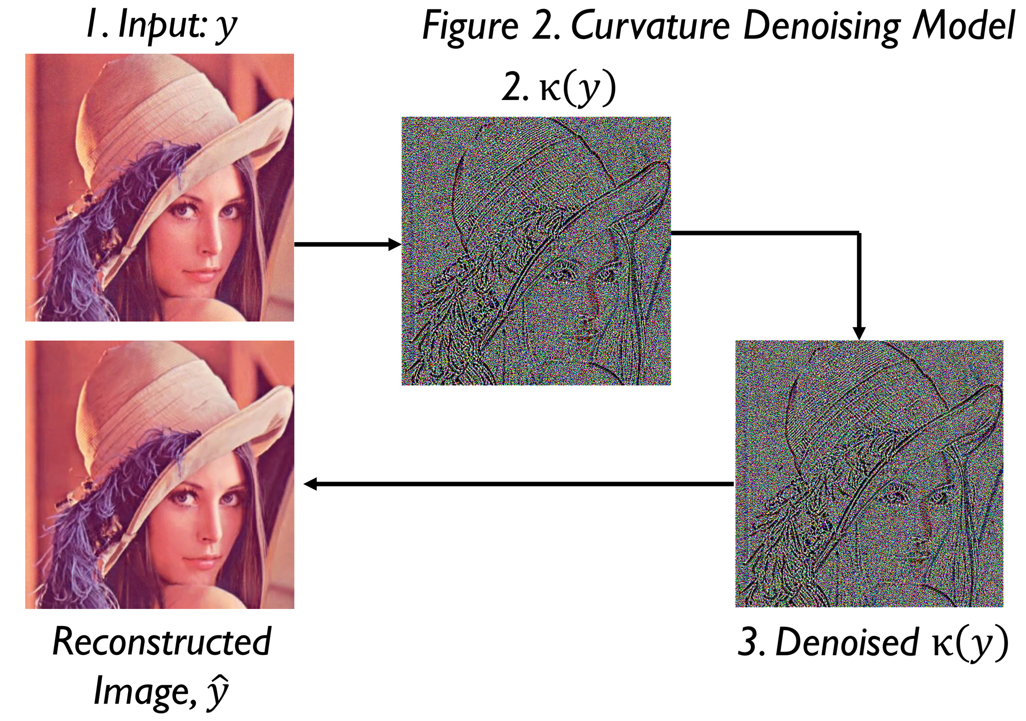
It was proposed by Bertalmío and Levine that an image could be denoised by denoising the curvature of its level lines and then reconstructing the original image from the denoised curvature, Fig. 2. The curvature of the level lines of an image y, $\kappa(y)$, can be defined as

$$\kappa(y) = \nabla \cdot \left(\frac{\nabla y}{|\nabla y|}\right).$$

The work in [1] considers curvature denoising because the curvature image is less effected by noise than the original image and it has been shown that a surface can be perfectly reconstructed from its level sets. Therefore, if we view an image as a surface, it can be perfectly reconstructed.

Our goal is to exploit self-similarity to denoise geometric information and reconstruct a clean image.

Figure 2. Curvature Denoising Model



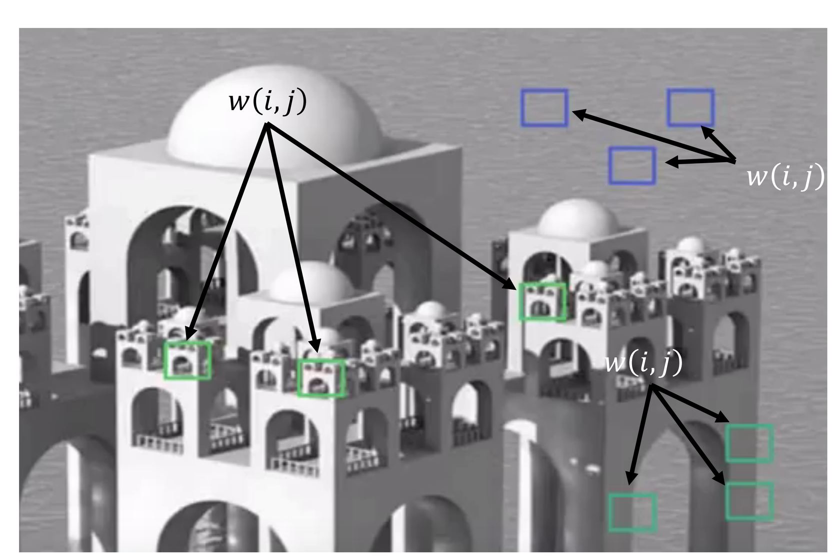
III. Non-local Means Model

Non-local means (NLM) [2] is a simple model that can be adapted for application to denoising in step 3 of Fig. 2 to gauge the best representation for our data. This method of image denoising is based on exploiting self-similarity of patches found in natural images. For a noisy discrete image y, the NLM approximation at pixel i is calculated from a weighted average of pixels in the image,

$$NLM[\hat{y}](i) = \sum_{j \in I} \omega(y(i), y(j))y(j),$$

where the set of weights $\{\omega(y(i), y(j))\}_j$ depend on the similarity between image patches located at pixels i and j with $0 \le \omega(y(i), y(j)) \le 1$ and $\sum_{j \in I} \omega(y(i), y(j)) = 1$. See Fig. 3 for an example.

Figure 3. Example of NLM



Each group of three boxes represents patches that were found to be similar within NLM.

IV. Experiments

Instead of running NLM on a natural image, we compute $\kappa(y)$ as shown in Fig. 2 and run NLM on the this image with different techniques for determining $\omega(y(i), y(j))$. Within the NLM framework, we attempted to exploit self-similarity by computing $\omega(y(i), y(j))$ using three different representations of the data.

The three representations we considered were:

1. Natural image data, y
2. Curvature data, $\kappa(y)$
3. Edge data, $e(y) = \frac{\nabla y}{|\nabla y|}$

To denoise in this way, we first compute $\kappa(y)$ and transform y according to one of the three representations above. We then use this representation for computing $\omega(y(i), y(j))$ at each pixel i.

As an example, an analysis of the weight metrics for the highlighted region shown in Fig. 4 for each representation is displayed in Fig. 5. Then we denoise $\kappa(y)$ by computing $NLM[\kappa(y)](i)$ for each i. Then we can reconstruct an estimated clean image from this result.

Figure 4. Images with Highlighted Patch

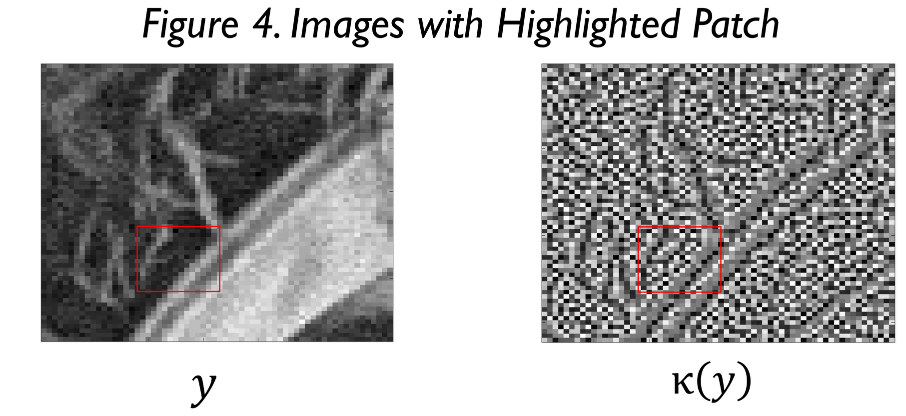
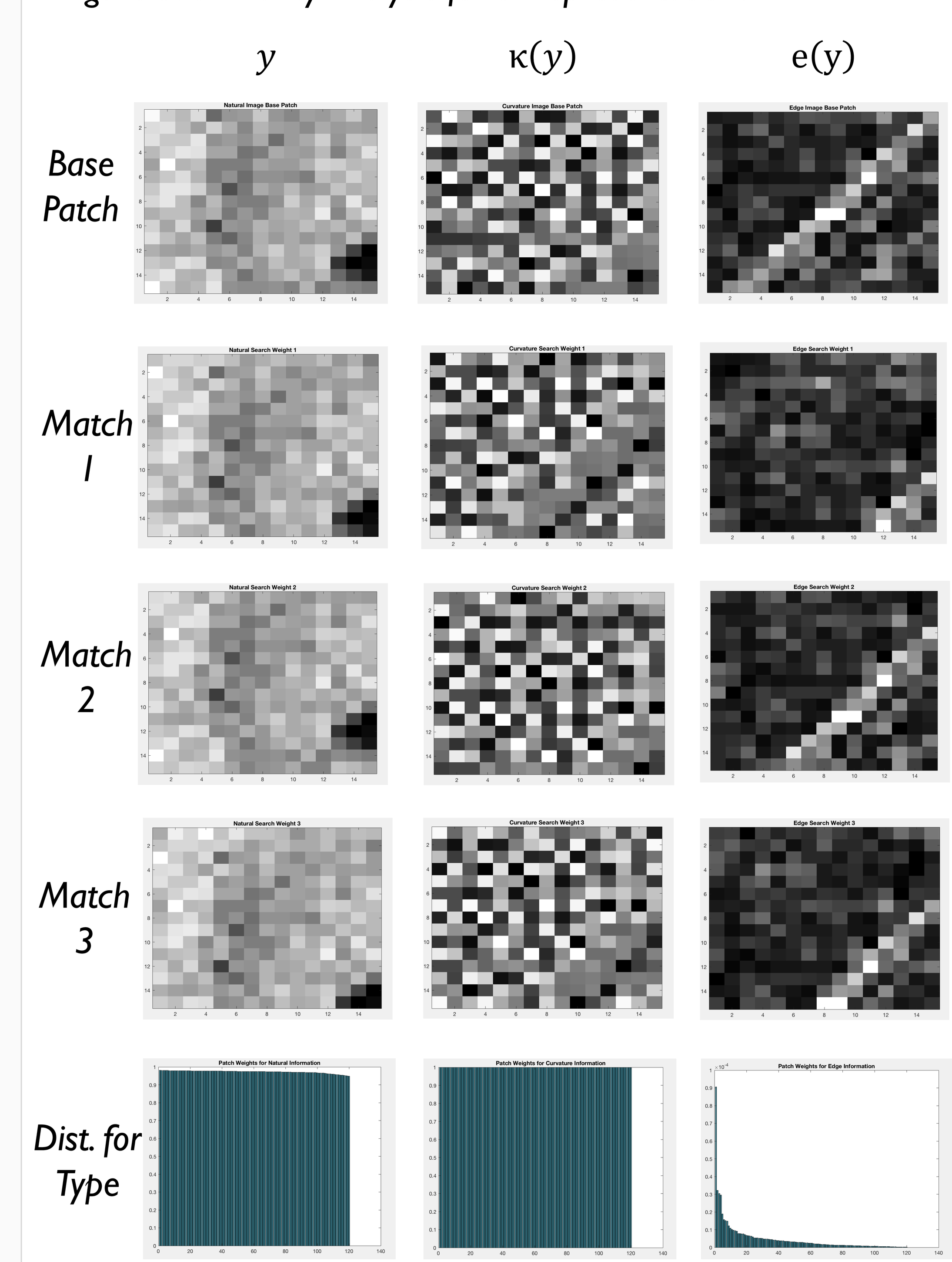


Fig. 5 displays the top three most similar patches for each base patch and for each representation. In the fifth row is a bar graph of the distribution of similarity scores of all patches found with a score greater than zero.

Figure 5. Similarity Analysis for 3 Representations



V. Results

We applied NLM to the curvature image, k(y), for Step 3 in the Fig. 2 pipeline using

$$NLM[\kappa(\hat{y})](i) = w_{i,j}\kappa(y(j)).$$

where the weights, $w_{i,j}$ are computed from y, $\kappa(y)$, and $e(y)$. The distributions in Fig. 5 suggest there is more self-similarity within y and $\kappa(y)$, but Fig. 6 indicates that the edge data, $e(y)$ produces weights that better preserve the geometric structure of the curvature image.

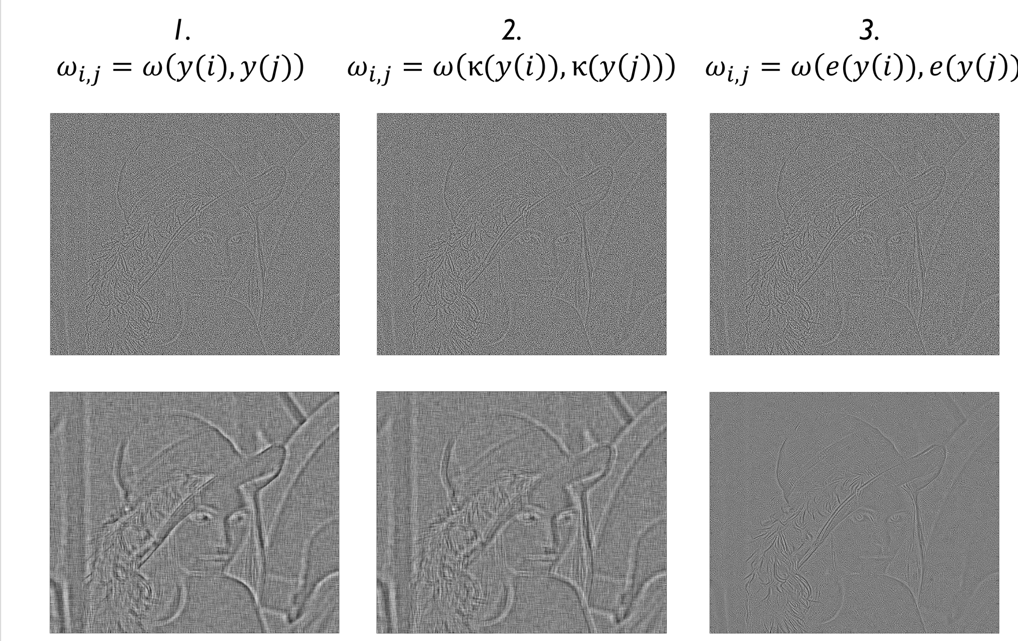


Figure 6. Results with each representation. Top Row, Noisy Curvature Image, σ=15. Bottom Row, Denoised Result.

VI. Future Work

While the edge based weights are promising with respect to preserving geometry, none of the three representations for self-similarity preserved contrast. Therefore more analysis needs to be performed to understand how self-similarity is effecting the quality of the denoised curvature result.

Additionally, it is possible that an optimal mechanism for measuring self-similarity which is used to compute the weights $\omega(y(i), y(j))$has yet to be considered. In the future, we plan to investigate other representations of image data aside from edge information, curvature information, and natural image information. Ultimately, our results demonstrate that a balance between self-similarity and geometry should result in the most effective denoising method.

VII. References

1. M. Bertalmío and S. Levine, "Denoising an Image by Denoising Its Curvature Image", *SIAM J. Imaging Sciences*, Vol. 7, pp. 187-211 (2014).
2. A. Buades, B. Coll, JM. Morel, "A Non-local Algorithm for Image Denoising", *Proceedings of the IEEE Computer Society Conference on Computer Vision and Pattern Recognition,* Vol. 2, pp. 60-65 (2005).
3. M. Lebrun, "An Analysis and Implementation of the BM3D Image Denoising Method," *Image Processing On Line*, pp. 175–213 (2012). https://doi.org/10.5201/ipol.2012.l-bm3d

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